Optimum Design of Disk Structures under Planar Loads

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Abstract

Disk structures under planar loads are commonly found in machines, such as disk brakes, automobile wheels, gears, etc. Weight reduction of such parts reduces inertia of the systems which helps in improving overall performance of the machines. In this research, a numerical method employing a stress-based material distribution scheme is utilized in the design of disk structures. During the process, stress distribution is calculated by finite element method. Then, elements with low stress are successively removed. The iteration process continues until the optimum topology is revealed. It is observed that the optimum topology follows the pattern of the corresponding principal stress trajectories. The resulting designs for different conditions provide a basic guideline for the optimum topology which can serve as a starting point for creating solid models for optimum shape design of disk structures.

Keywords: optimumtopology design, disk brakes, stress based material distribution

1. Introduction

Optimum design principles have been applied to the design of mechanical parts for decades. Much research is conducted on development of an efficient numerical algorithm for optimum geometric design of freeform mechanical parts. The major approaches to solving complex optimum geometric design problems can be classified as (i) size optimization and (ii) topology optimization. The former is to represent the geometry or boundary with a mathematical relationship and alter the control parameters based on gradient information to obtain the optimum shape. This approach is popular in fluid mechanics related problems where the relationship between geometry and flow behavior is highly non-linear and non-local, i.e. a minimum drag body design [1, 2]. The latter, topology optimization, operates on material placement, orientation, or distribution. The simplest way is to distribute the material (removing or adding) , according to the local constrained condition, i.e., stress value [3]. This approach is very effective in design of complicated structural parts, where optimum topology must be achieved before performing size optimization.

This research uses the second approach. A stress based material distribution method is utilized in the design of disk structures under planar load. Various cases of hollow disks under planar loads were studied. There is no unique solution to topology optimization, as different preferences and parameters used, such as element removal rate and mesh size, can lead to different solution, which can be considered as Pareto solutions. The simplest forms of solutions shall be considered optimum in this research. Optimum results were studied in-depth to gain understanding of the underlining physics. In the end, a design guideline is proposed for a range of disk structure with different proportion and different number of securing holes. The guideline is applicable to design of disk structures such as disk brakes, wheels, chain sprockets, gears, etc.

2. Stress based material removal algorithm

Variations of stress based material distribution algorithms have been utilized in structural design problems [4, 5]. Minimizing material usage with stress based algorithm yields equivalent results for the more complicated stiffness based approaches [6]. This research utilizes an intuitive stress based algorithm adapted from the idea of tree growth [7], in an inverse sense. The algorithm has been proven to work well on planar structures [8]. A modification is done to the algorithm from [8] to trigger element removal from a finite element model. The algorithm works in the following steps:

Step 1) Create a finite element model of a blank is created

Step 2) Obtain solution for stress distribution

Step 3) Compute the required material thickness index t_i^* of each element *i* with

$$t_i^* = t_i\,[r\,(\sigma_i/\sigma_{max})^p + (1 - r)] \tag{1}$$

where

t_i = element thickness,

r = relaxation factor (use a value less than 1 to slow down the process),

σ_i = von Mises stress of element *i*,

σ_{max} = maximum von Mises stress on the part,

p = exponent, between 0.5 and 1, used to adjust sensitivity of interaction between stress and thickness (a lower value helps finding topology at low stress area, but decreases convergence rate).

Step 4) Set a thickness index threshold t_{set} as shown below and turn-off elements (set thickness toward zero), which have $t_i^* < t_{set}$, and keep the rest of the elements intact.

$$t_{set} = t_{min}^* + u\,(t_{max}^* - t_{min}^*) \tag{2}$$

where

t_{min}^* = minimum thickness index found from step 3),

t_{max}^* = maximum thickness index found from step 3),

u = a number between 0 and 1 representing element cut-off point.

Step 5) Repeat from Step 2) until the topology is revealed or the domain is separated.

3. Optimum design of disk structures

The algorithm described in section 2 was utilized in the design of disk structures under planar loads.

3.1 Design conditions

A hollow disk structures under a planar load in the form of torsion is shown in Fig. 1. Torsion load is applied through uniformly distributed tangential forces on every node points on the outer rim of the disk. There are three rigid constraints on the inner edge (*n* = 3) represented by solid dots. These points represent bolts in real applications. Elements on outer and inner edges of the disk are not to be removed. Thickness of the plate is set at 1 unit. Calculations are done on a dimensionless basis, in order to keep the focus on topology only. The study covers 12 different configurations obtained by varying numbers of equally-spaced constrain points, *n*, from 3 to 6 and varying the ratio between outer and inner radius (r_o/r_i) of the disk at 2, 3 and 4.

3.2 Optimum design of a disk structures with three constrained points under torsion

A disk with three constrained points and r_o/r_i = 3 is used as an example in this section to demonstrate the design process and to perform in-depth analysis. Parameters used in the algorithm are r = 0.9, u = 0.1 and p = 1. Those parameters correspond to a less aggressive cutting scheme which makes convergence slow but they prevent overcutting. A quadrilateral finite element model is used with 81 by 28 elements. Fig. 2 shows reduction of volume with iteration numbers.

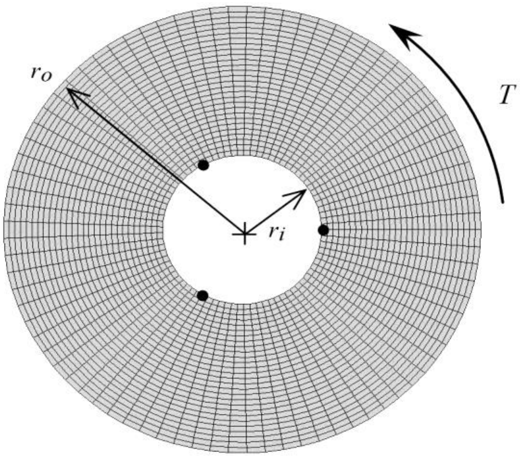


Fig. 1.A finite element model of a hollow disk under torsion.

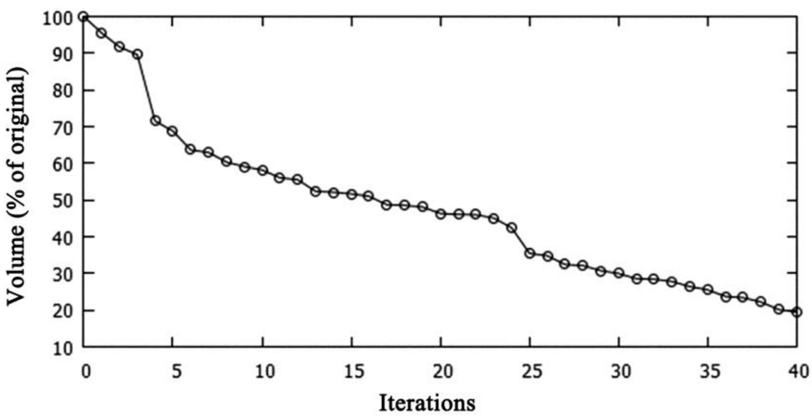


Fig. 2. Volume vs. iterations.

Fig. 3 shows results of element removal which gradually converges to the optimum topology at iteration 35, and domain separation occurs after iteration 35. The optimum topology can be used to construct a solid model, as shown in Fig. 4 (a), for example. Simulation results shows nearly uniform stress distribution on the spokes with very minor signs of stress concentration around the fillets. A small geometric adjustment can cut more weight and perfect the design. Fig. 4 (b) shows stress distribution of an arbitrary designed disk structure with similar weight under the same load. Although stresses on the spokes appear well distributed, high stress concentration can be observed around the hub area, due to the bending effect.
The optimum design has 10% less stress and 42% less deflection.

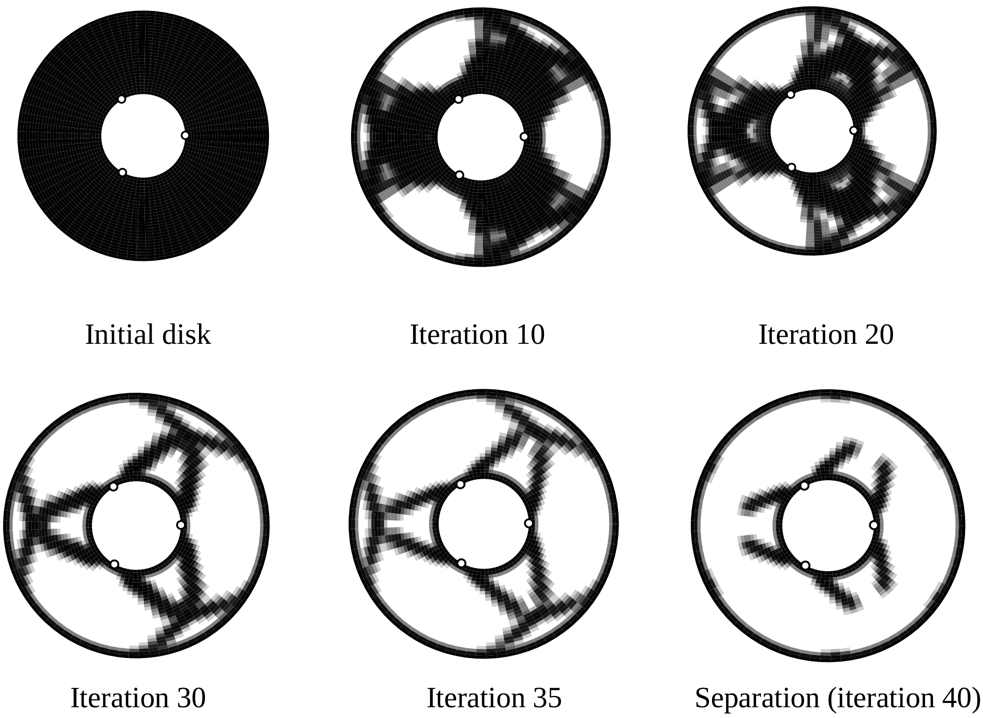


Fig. 3. Convergence to the optimum topology.

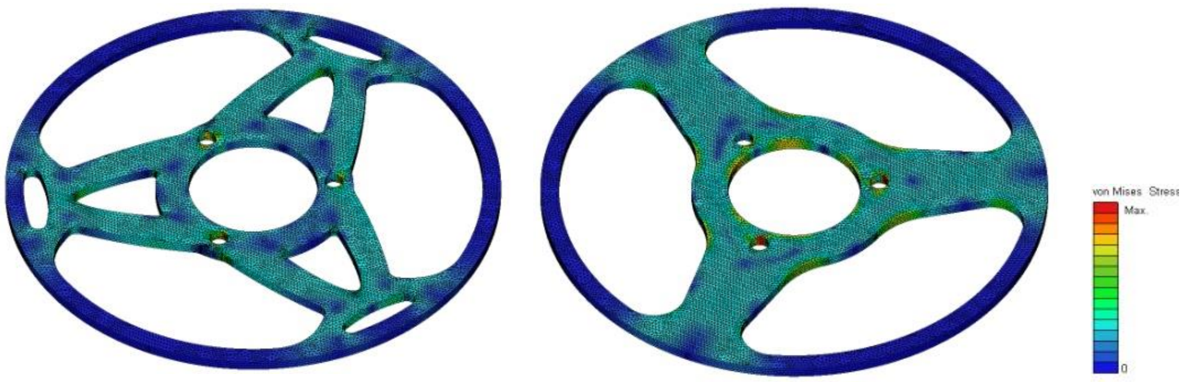
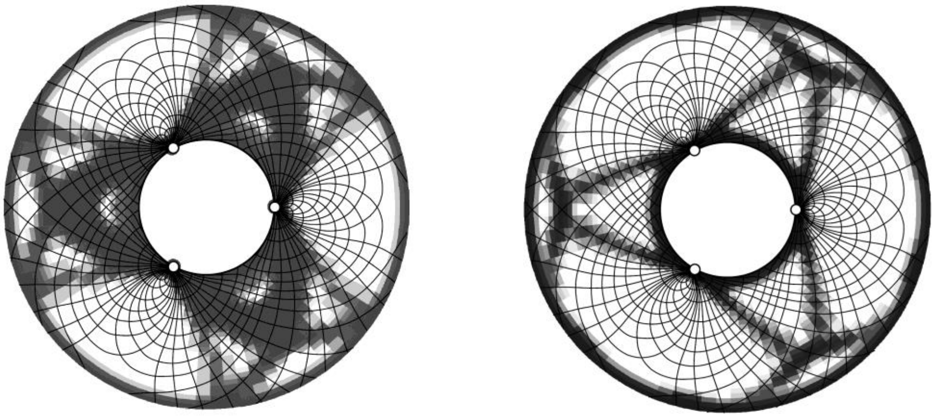


Fig. 4. Stress simulation of disk structures with optimum topology under torsion.
(a) Optimum disk. (b) Arbitrary designed disk.

3.3 Observation

It can be observed from Fig. 3 that optimum topology is not unique. If one decides to stop the process at iteration 20, a more complicate topology, different from that of the final one, can also be a solution.

Principal stress trajectories of the initial hollow disk in section 3.2 are numerically computed and compared to the optimum topology obtained previously. The comparison in Fig. 5 shows that topology of designs from iterations 20 and 35 that follow the pattern found in principle stress trajectories of the initial hollow disk. Furthermore, it is observed
from Fig. 6 that the designed structures of the disk lie in the area corresponding to high initial stress.



(a) **(b)**

Fig. 5. Comparison of principle stress trajectories of the initial disk to(a) topology from iteration 20 and (b) final topology from iteration 35.

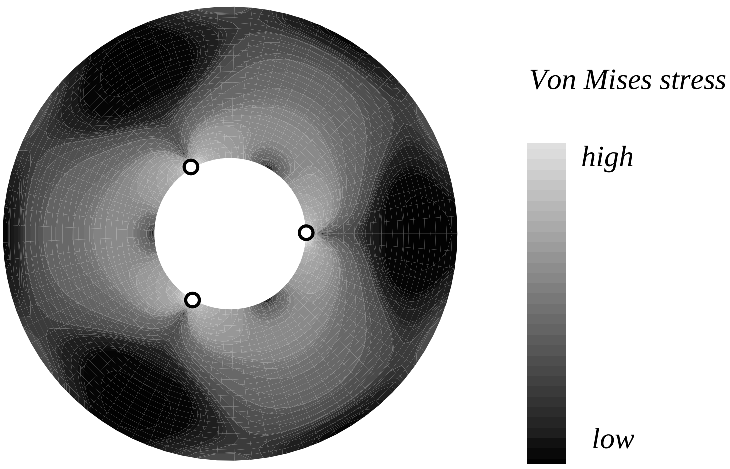


Fig. 6. Stress distribution in the initial disk.

3.4 Other load types

Two more load cases have been explored: radial load and mixed load. Note that mixed load is created by applying tangential and normal forces of the same magnitude on every node on the outer rim. In each load case, varying the convergence rate by using different values of *u* and *p* affected the final topology. The resulting designs share some common characteristics that are observed in Fig. 7 and 8. In all designs, the areas opposite the constrained point toward the outer rim are blank and there are two main spokes coming out of each constrained point. The common characteristics found here are also possessed by the optimum topology for torsion load, hence the next section presents the results for optimum topology under torsion load, under the assumption that they can be adapted to other load conditions.

4. Design guideline for disk structures under torsion

Following the design process discussed earlier, 12 cases of disk structures with different r_o/r_i and different numbers of constrain points have been investigated. Numbers of elements used around the disk are varied between 78 and 85 elements to ensure that constrained points align with the nodes. The resulting designs are summarized in Table 1 to provide a guideline for designing disk structures. Overall, the optimum design can be described as a set of *n* two-spoke structures with each pair connected to each constrained point, where *n* is the number of constrained points. More complicated topology is observed in the case of 6 constrained points. The guideline can provide a starting point for detailed designing of disk structures.

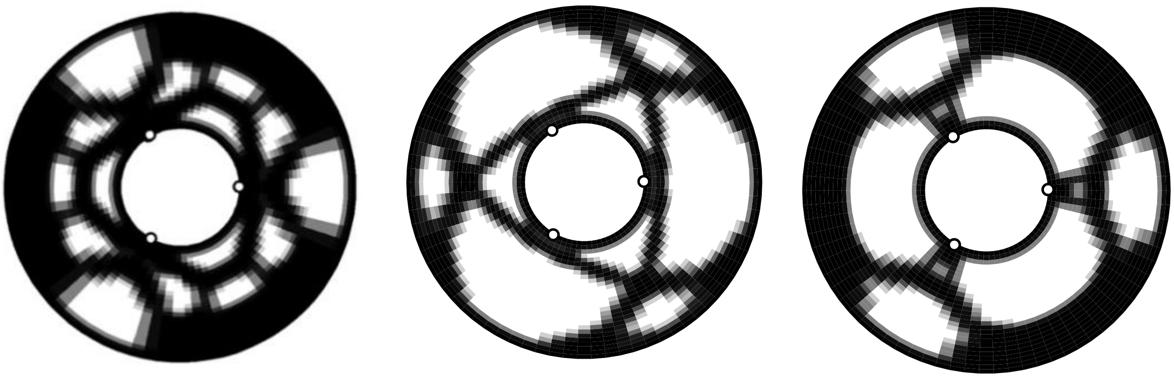


Fig. 7. Possible optimum topology for disk structures under radial load.

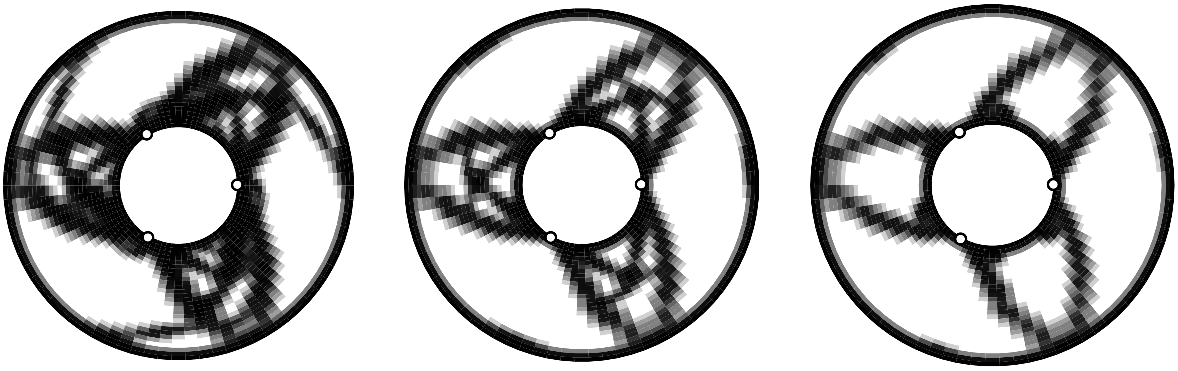
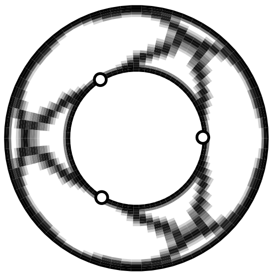
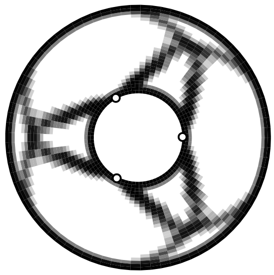
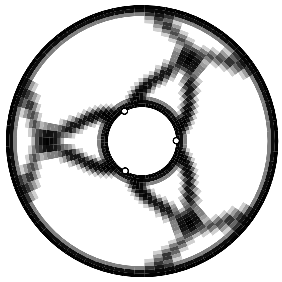
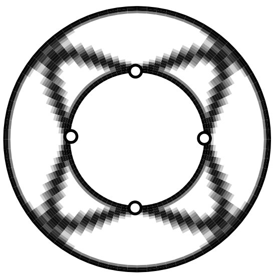
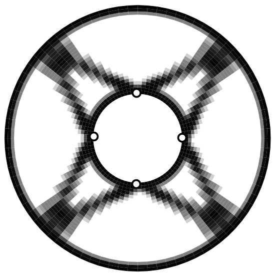
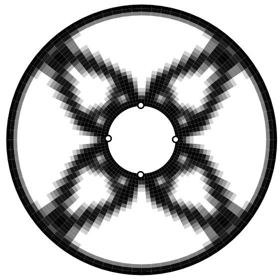
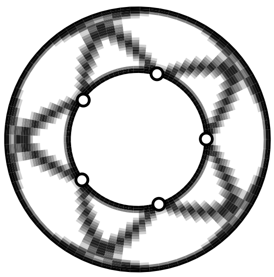
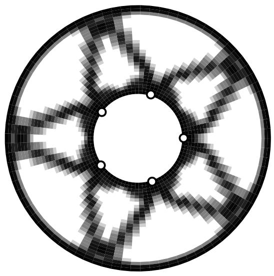
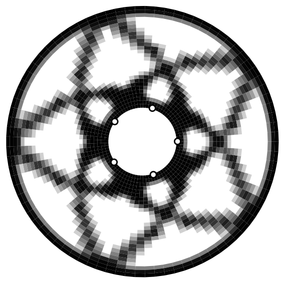
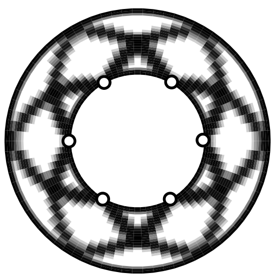
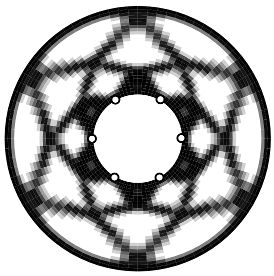
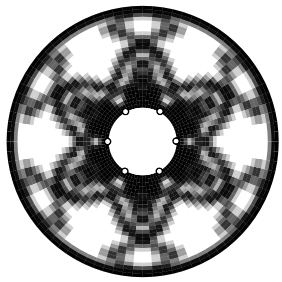


Fig. 8. Possible optimum topology for disk structures under mixed load.

Table1. Design guideline for disk structures under torsion.

r_o/r_i		
2	**3**	**4**
		
		
		
		

5. Conclusion

A stress-based material distribution algorithm has been implemented in finding optimum topology of disk structures under planar load. The algorithm itself is very simple yet effective. The optimum topology of disk structures under torsion load obtained has been compared to the principal stress trajectories and the von Misesstesss distribution of the full plate disk. The comparison showed good agreement. Optimum topology under other load conditions, radial and mixed load, are found to have some common characteristics to the case of torsion.

During the iteration there exist some designs that can serve as alternative solutions to the problem. Moreover varying design parameters also alters the solutions. This is common for topology optimization where a set of Pareto solutions can be expected. However the final designs selected here are the simplest ones. They are presented in Table 1 for 12 different combinations of r_o/r_i ratios and numbers of constrained points. Table 1 can be used as a starting points for the optimum design of disk structures for various engineering applications.

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